't Hooft anomaly matching condition and Chiral symmetry breaking without fermion bilinear condensate

Satoshi Yamaguchi (Osaka University)

(arXiv:1811.09390)

Introduction

Strongly coupled gauge theory is interesting

- Theoretical (mathematical) interest
- QCD
- Beyond the standard model
  - Technicolor
  - Composite Higgs models
  - …

Some non-perturbative techniques are needed

a nonperturbative technique

’t Hooft anomaly matching condition

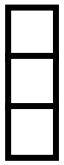
Advantage: “Rigorous” (in physicists sense)

Disadvantage: “Qualitative”

Recently there has been a new progress (including higher form symmetry)

[Gaiotto, Kapustin, Seiberg, Willett 14],
[Gaiotto, Kapustin, Komargodski, Seiberg 16]

I explored various gauge theories and find an interesting example.

4 dim SU(6) with a Weyl fermion in 

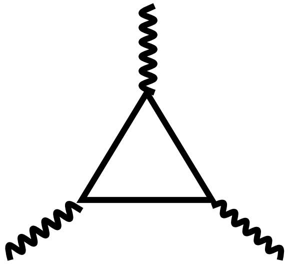
Chiral symmetry is spontaneously broken but $\langle\psi\psi\rangle = 0$

Do not confuse!

Three different “anomalies”

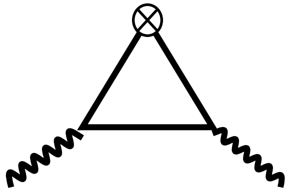
1 Gauge anomaly

Inconsistency of the theory



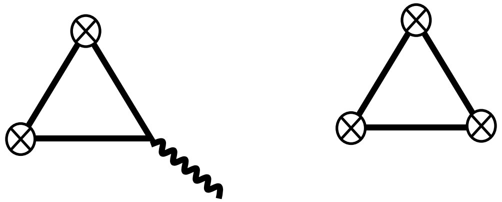
2 Anomaly for a global symmetry

Non-existence of a global symmetry
which exists in the classical theory



3 ’t Hooft anomaly

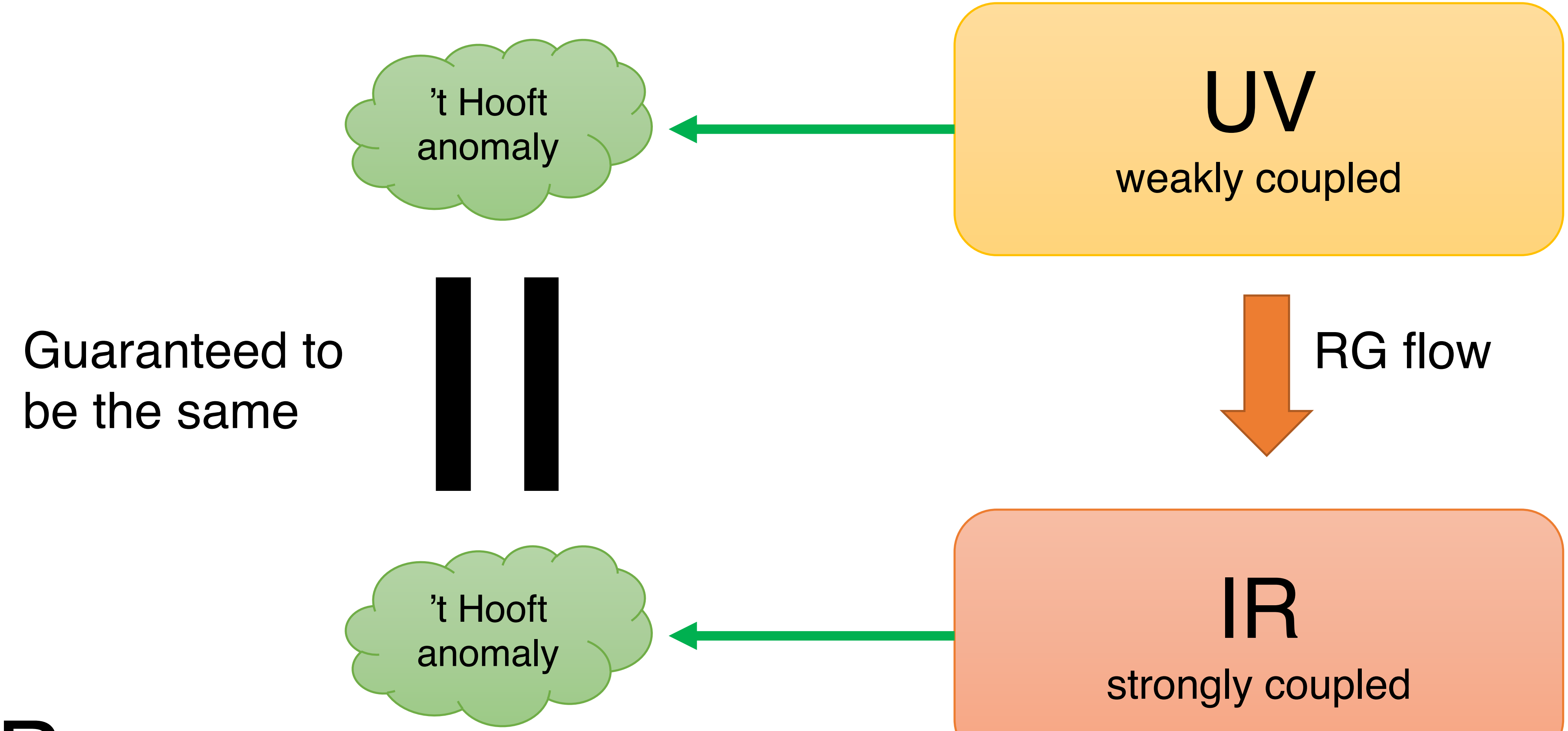
Useful tool to study the theory



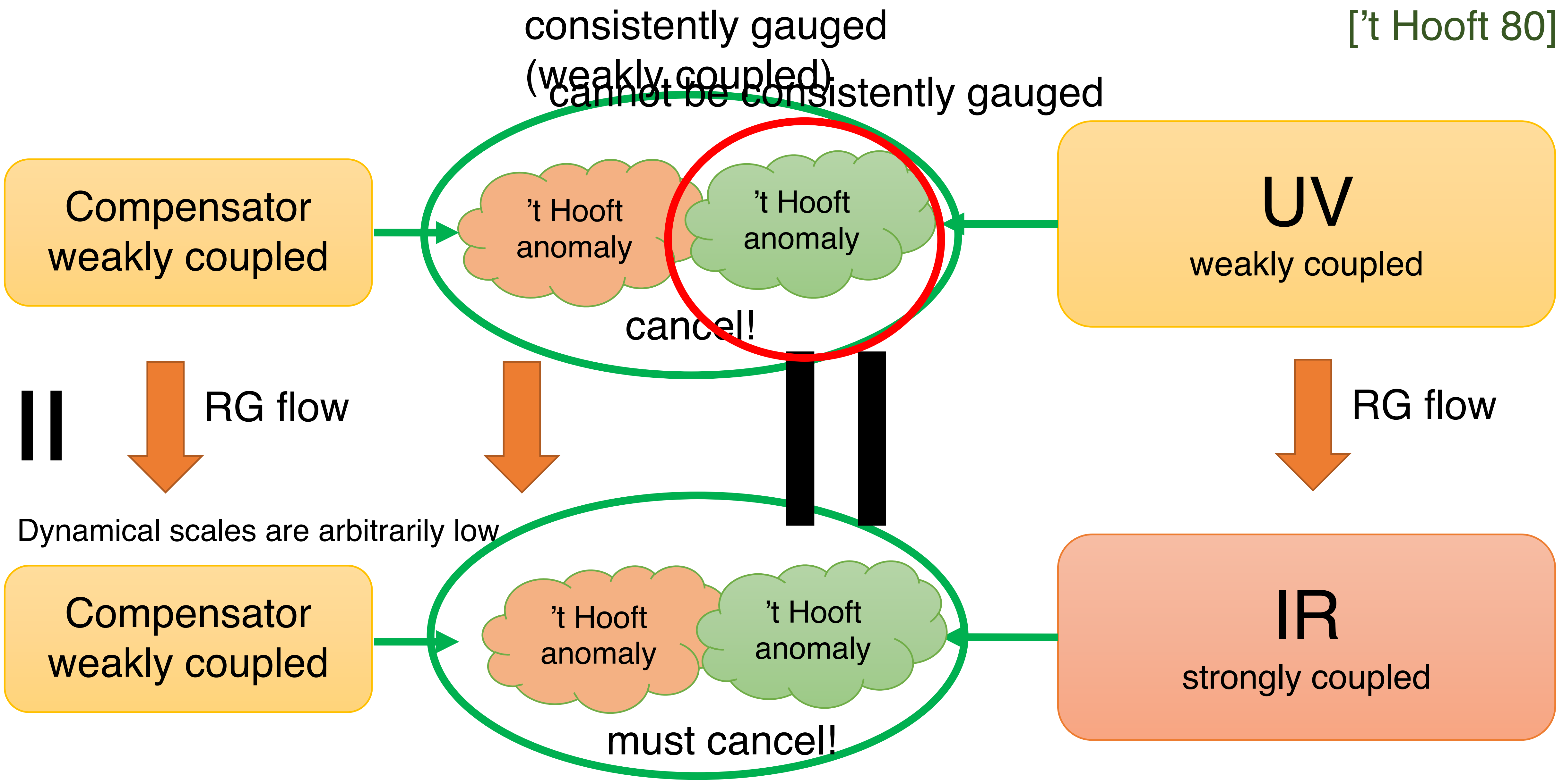
3 't Hooft anomaly matching condition

['t Hooft 80]

anomaly when background gauge field for a global symmetry is introduced



Because ...



consistently gauged
(weakly coupled)
cannot be consistently gauged
['t Hooft 80]
Compensator
weakly coupled
't Hooft
anomaly
't Hooft
anomaly
cancel!
UV
weakly coupled
RG flow
RG flow
Dynamical scales are arbitrarily low
Compensator
weakly coupled
't Hooft
anomaly
't Hooft
anomaly
must cancel!
IR
strongly coupled

Plan

- Introduction
- The model and the symmetry
- Analysis by ’t Hooft anomaly matching condition
- An example --- chiral symmetry breaking without bilinear condensate
- Summary and discussion

The model and symmetry

4 dim SU(N) gauge theory

A massless Weyl fermion in an irreducible representation R

ψ_α^I $\quad I = 1, \ldots, \dim R$ gauge index

$\alpha = 1, 2$ spinor index

$$S = \int d^4x \left[-\frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} + i\bar{\psi}_{I\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} (\partial_\mu \delta^I_J - i A^a_\mu T_a{}^I{}_J) \psi^J_\alpha \right]$$

1 Gauge anomaly

R is real or pseudo-real ➡ No perturbative gauge anomaly

$$\mathrm{Tr}_R[T_a\{T_b, T_c\}] = 0$$

$N > 2$ ➡ No global gauge anomaly of [Witten 83]

Remark: there is still possibility to have unknown gauge anomaly,
unless you find non-perturbatively gauge invariantly regularized (lattice) theory.

Chiral symmetry

$$S = \int d^4x \left[-\frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} + i\bar{\psi}_{I\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} (\partial_\mu \delta^I_J - i A^a_\mu T_a{}^I{}_J) \psi^J_\alpha \right]$$

$$\psi \to e^{i\alpha} \psi$$

The action is invariant.

2 But the path integral measure is not invariant.

$$\int D\psi D\bar{\psi} \to \int D\psi D\bar{\psi} e^{i\alpha \ell \nu}$$

$$\nu := \frac{1}{8\pi^2} \int \mathrm{Tr}_\square [F \wedge F]$$

integer called “instanton number”

$$\mathrm{Tr}_R [T_a T_b] = \ell \, \mathrm{Tr}_\square [T_a T_b]$$

ℓ : integer called “Dynkin index” of R

$$\int D\psi D\bar{\psi} \to \int D\psi D\bar{\psi} e^{i\alpha\ell\nu}$$

If $\alpha = \dfrac{2\pi n}{\ell}$ the measure is also invariant.

$n \in \mathbb{Z}$

⇒ Even quantum mechanically, $\mathbb{Z}_\ell$ symmetry

$\psi \to e^{2\pi i n/\ell}\psi$ exists.

Let us call this symmetry "chiral symmetry"

Problem: Is this chiral symmetry $\mathbb{Z}_\ell$ spontaneously broken?

Key idea: Center symmetry

eg. SU(N) pure Yang-Mills theory on lattice

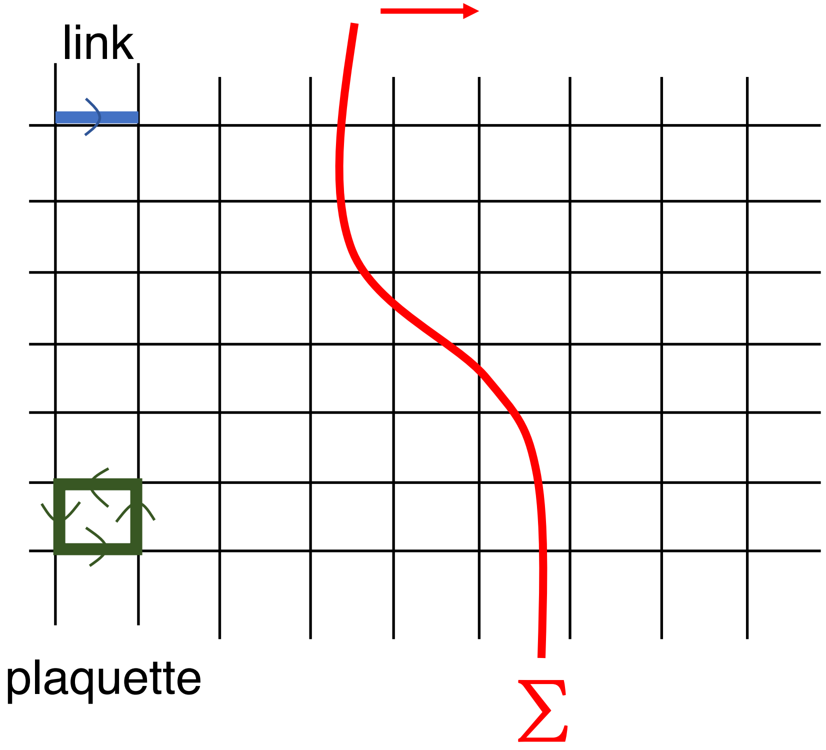
Gauge field = a group element at each link r

$$U_r \in SU(N)$$

The action = sum of plaquettes

Choose oriented codimension 1 surface Σ

$$e^{2\pi i k/N} \in \mathbb{Z}_N \subset SU(N)$$



Transformation

$$U_r \longrightarrow U_r e^{2\pi i k I/N}$$

I : intersection number of Σ and r

→ plaquettes are invariant

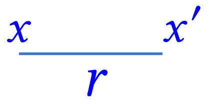
→ The action is invariant

Center symmetry $U_r \longrightarrow U_r e^{2\pi i k I/N}$

Remarks

- The center symmetry is NOT the gauge symmetry.

cf. gauge symmetry $U_r \longrightarrow g_x U_r g_{x'}^{-1}$

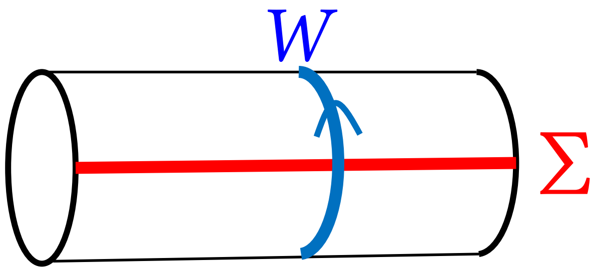


- The center symmetry is a 1-form global symmetry.

[Gaiotto, Kapustin, Seiberg, Willett 14]

- A fundamental Wilson loop may have charge of the center symmetry

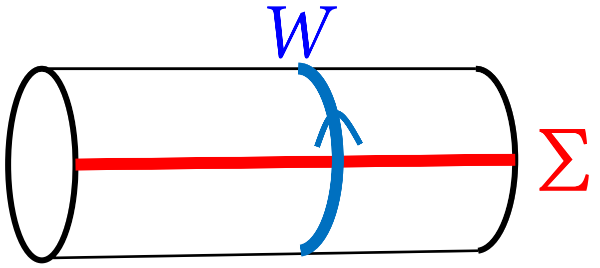
$$W \longrightarrow W e^{2\pi i k/N}$$



Center symmetry $U_r \to U_r e^{2\pi i k I/N}$

- A fundamental Wilson loop may have charge of the center symmetry

$$W \to W e^{2\pi i k/N}$$



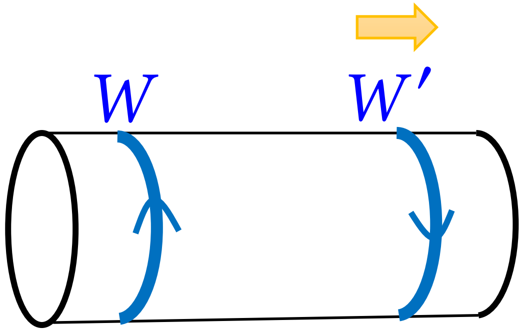
If $\langle W \rangle \neq 0$ the center symmetry is spontaneously broken

Coulomb law or perimeter law

$$\langle W \rangle = 0$$

Area law

send this one to infinity



$$\langle W W' \rangle \to \langle W \rangle \langle W' \rangle$$

Confinement $\Longleftrightarrow$ The center symmetry is NOT spontaneously broken

Fermion and center symmetry

Introduce ψ in rep R → $R(U_r)$ appear in the action

→ Center symmetry is broken

How ?

c "N-ality" (num. of boxes in the Young diagram) mod N

$$R(U_r) \to R(U_r e^{2\pi i k/N}) = R(U_r)\underbrace{e^{2\pi i k c/N}}$$

If this=1 it is still a symmetry

c "N-ality" (num. of boxes in the Young diagram) mod N

$$R(U_r) \to R(U_r e^{2\pi i k/N}) = R(U_r)e^{2\pi i k c/N}$$

If this=1 it is still a symmetry

$q = \gcd(N, c)$ $\mathbb{Z}_q \subset \mathbb{Z}_N$ still remains.

We concentrate on $q > 1$

Confinement ⟺ The center symmetry is NOT spontaneously broken

still holds

Summary of our model

4 dimensional SU(N) gauge symmetry with a Weyl fermion in rep R

Chiral symmetry $\mathbb{Z}_\ell$ 0-form symmetry (usual symmetry)

ℓ : Dynkin index of R

Center symmetry $\mathbb{Z}_q$ 1-form symmetry

$q = \mathrm{gcd}(N, c)$ c : N-ality of R

Confinement ⟺ The center symmetry is NOT spontaneously broken

Plan

- Introduction
- The model and the symmetry
- Analysis by ’t Hooft anomaly matching condition
- An example --- chiral symmetry breaking without bilinear condensate
- Summary and discussion

Analysis by ’t Hooft anomaly matching condition

strategy

Chiral symmetry $\mathbb{Z}_\ell$ 0-form symmetry (usual symmetry)

We want to argue the spontaneous symmetry breaking

Center symmetry $\mathbb{Z}_q$ 1-form symmetry

Make use of mixed 't Hooft anomaly matching between these symmetries.

3

Introduce a gauge field background for the center symmetry and see if the chiral symmetry is broken.

In the background center symmetry gauge field

Chiral symmetry transformation $\psi \to e^{2\pi i n/\ell}\psi$

$$\int D\psi D\bar{\psi} \to \int D\psi D\bar{\psi} e^{2\pi i n \nu}$$

We will discuss in detail later if we have time

$\nu := \frac{1}{8\pi^2}\int \mathrm{Tr}_{\square}[F \wedge F]$ may not be an integer in this background

but $\nu \in \frac{1}{q'}\mathbb{Z}$ $\quad \frac{N}{q^2} = \frac{N_0'}{q'}$ irreducible fraction

Only $\mathbb{Z}_{\ell/q'} \subset \mathbb{Z}_\ell$ is the symmetry in this background

Only $\mathbb{Z}_{\ell/q'} \subset \mathbb{Z}_\ell$ is the symmetry in this background

This is the ’t Hooft anomaly

't Hooft anomaly and spontaneous symmetry breaking

(Back to the original theory)

Assume confinement (gapped and center symmetry preserved) and the chiral symmetry is preserved (not spontaneously broken).

We have a gapped vacuum invariant under the center and the chiral symmetry.

In the low energy limit, ‘t Hooft anomaly cannot be reproduced, since the low energy effective theory is empty.

This contradict to the fact that ’t Hooft anomaly is RG invariant.

The assumption (confinement and chiral symmetry preserved) is wrong!

The result

If confinement, the chiral symmetry $\mathbb{Z}_\ell$ must be broken at least to $\mathbb{Z}_{\ell/q'}$

$$\frac{N}{q^2} = \frac{N_0'}{q'}$$ irreducible fraction

$$q = \gcd(N, c)$$

c "N-ality" (num. of boxes in the Young diagram) mod N

Instanton number in the center symmetry gauge field background

We use the formulation of

[Gaiotto, Kapustin, Komargodski, Seiberg 16]

Introduce the center symmetry gauge field and see $\nu := \frac{1}{8\pi^2}\int \mathrm{Tr}_{\square}[F \wedge F]$

Two unfamiliar issues

- To gauge a 1-form symmetry ⇨ gauge field is 2-form
- To gauge a discrete symmetry ⇨ let me explain a little bit

Warm up: to gauge a usual discrete symmetry.

Idea: introduce a U(1) gauge field and break it to $\mathbb{Z}_q$ by Higgs mechanism.

A U(1) gauge field H charge q scalar

$$S = \int d^4x D_\mu H^\dagger D_\mu H + \cdots \qquad D_\mu H := \partial_\mu H - iqA_\mu H$$

If H condensate, U(1) gauge symmetry is broken to $\mathbb{Z}_q$ gauge symmetry

This is what we want!

$$H = he^{i\phi} \qquad\qquad \phi \sim \phi + 2\pi$$

$$S = \int d^4x h^2(\partial_\mu \phi - qA_\mu)^2 + \cdots$$

Higgs mass and vector particle mass → ∞ limit

$$\partial_\mu \phi - qA_\mu = 0$$

If $q = 1$, this constraint implies A is pure gauge and nothing remains.

But $q > 1$, it is not completely pure gauge and something remains.

Eg. Wilson loop $e^{i\int A}$ is not always 1 but q-th root of 1.

We obtain

$\mathbb{Z}_q$ gauge field = (A, ϕ)

- A U(1) gauge field
- $\phi \sim \phi + 2\pi$ scalar

constraint $qA = d\phi$

Increase rank of the forms by 1

$\mathbb{Z}_q$ 2-form gauge field = (B, C)

- B U(1) 2-form gauge field
- C U(1) gauge field

constraint $qB = dC$

$\mathbb{Z}_q$ 2-form gauge field = (B, C)

B U(1) 2-form gauge field

C U(1) gauge field

constraint $qB = dC$

Gauge symmetry parameter λ U(1) gauge field

$$B \longrightarrow B + d\lambda \qquad C \longrightarrow C + q\lambda$$

Remark:

Wilson surface $e^{i \int_{\text{2-cycle}} B}$ is q-th root of 1.

Coupling (B, C) to the SU(N) gauge field

A SU(N) gauge field

$\Rightarrow$ $\mathcal{A}$ U(N) gauge field whose traceless part is A

Kill this trace part by the λ gauge symmetry $\mathcal{A} \to \mathcal{A} + \lambda 1$

$$B \to B + d\lambda \qquad C \to C + q\lambda$$

The constraint is imposed. $\mathrm{tr}\mathcal{F} = NB$

SU(N) gauge field strength $F = \mathcal{F} - B1$ is λ gauge invariant.

Locally nothing changes, but some global topological effect remains.

Instanton number

$$\nu = \frac{1}{8\pi^2}\int \mathrm{tr}[F \wedge F]$$

$$= \frac{1}{8\pi^2}\int \mathrm{tr}[(\mathcal{F} - B1) \wedge (\mathcal{F} - B1)]$$

$$= \underline{\frac{1}{8\pi^2}\int \mathrm{tr}[\mathcal{F} \wedge \mathcal{F}]} - \frac{N}{8\pi^2}\int B \wedge B$$

an integer for a spin manifold

$$\nu = -\frac{N}{8\pi^2}\int B \wedge B \quad \mathrm{mod}\ 1$$

$$\nu = -\frac{N}{8\pi^2}\int B\wedge B \mod 1$$

Our B has Wilson surface $\frac{1}{2\pi}\int_{\text{2-cycle}} B \in \frac{1}{q}\mathbb{Z}$

Thus for a spin manifold

$$\nu \in \frac{N}{q^2}\mathbb{Z} = \frac{1}{q'}\mathbb{Z}$$

$\frac{N}{q^2} = \frac{N_0'}{q'}$ irreducible fraction

Remark:

- Chiral symmetry transformation $\psi \to e^{2\pi i n/\ell}\psi$

$$\int D\psi D\bar{\psi} \to \int D\psi D\bar{\psi}\, \underline{e^{2\pi i n \nu'}} \qquad \nu' = -\frac{N}{8\pi^2}\int B \wedge B$$

extra phase depending on
the external gauge field

- The background with non-trivial ν' can be realized as 4-torus with twisted boundary condition.

['t Hooft 79, 80], [Witten 00]

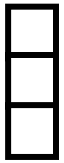
Plan

- Introduction
- The model and the symmetry
- Analysis by ’t Hooft anomaly matching condition
- An example --- chiral symmetry breaking without bilinear condensate
- Summary and discussion

An example

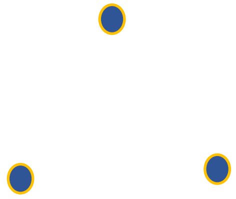
chiral symmetry breaking without bilinear condensate

Example: (It will be fun to consult tables in [Slanski 81], [Yamatsu 15])

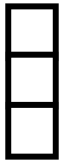
4 dim SU(6) with a Weyl fermion in 

$\ell = 6$ $c = 3$ $q = q' = 3$

If confinement, the chiral symmetry $\mathbb{Z}_6$ is broken to $\mathbb{Z}_2$

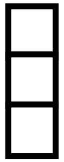


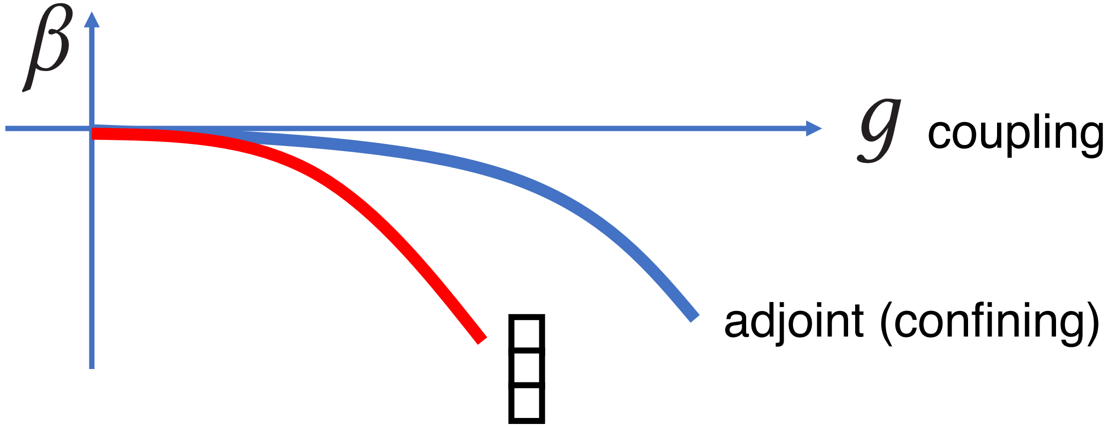
Confinement?

Example: 4 dim SU(6) with a Weyl fermion in 

(a bit speculative)

It is quite likely that this theory is confining.

Reason:  is “smaller” representation than adjoint → (coupling constant runs faster)
SU(6) with adjoint Weyl fermion is known to be a confining theory.



→ the chiral symmetry $\mathbb{Z}_6$ is broken to $\mathbb{Z}_2$

the chiral symmetry $\mathbb{Z}_6$ is broken to $\mathbb{Z}_2$

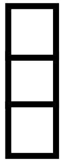
You may guess

$$\langle \psi\psi \rangle \neq 0$$

as usual.

But this cannot be true

$$\epsilon^{\alpha\beta}\psi_\alpha^I\psi_\beta^J B_{IJ} = 0$$ identically!

SU(6) invariant bilinear form
anti-symmetric for 

Plan

- Introduction
- The model and the symmetry
- Analysis by ’t Hooft anomaly matching condition
- An example --- chiral symmetry breaking without bilinear condensate
- Summary and discussion

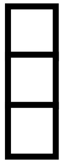
Summary and discussion

Summary:

- 4 dim SU(N) gauge theory with a Weyl fermion in irrep R

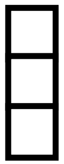
  Constraints on the chiral symmetry breaking is obtained

- An interesting example

  4 dim SU(6) with a Weyl fermion in 

  Chiral symmetry is spontaneously broken but $\langle\psi\psi\rangle = 0$

Discussion

4 dim SU(6) with a Weyl fermion in 

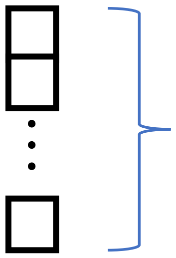
Chiral symmetry is spontaneously broken but $\langle \psi\psi \rangle = 0$

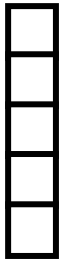
What causes the chiral symmetry breaking?

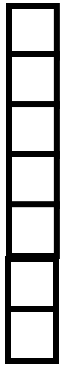
Maybe $\langle \psi\psi\psi\psi \rangle \neq 0$

Two possible 4-fermi operators

Other examples?

SU(4k+2) with  2k+1 has similar ’t Hooft anomaly and $\psi\psi = 0$ identically

SU(10) with  Confinement is not quite likely

SU(14) with  Not asymptotically free